STRESS RELAXATION, CREEP, AND UNIAXIAL STRAIN: GENERAL AND SPECIAL FEATURES

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Papers devoted to the development of methods of describing the behavior of materials during creep and stress relaxation in terms of uniaxial strain data are usually based on the idea of the existence of a mechanical equation of state, i.e., an equation relating deformation or rate of deformation, temperature, stress, and time for such processes. The idea of a mechanical equation of state was first put forward by Nadai [1] and Zener and Holloman [2]. Holloman [3] has reported some experimental evidence for the existence of such an equation of state. However, Orowan [4], Dorn et al. [5], and Johnson et al. [6] have obtained data which are not in agreement with this idea. Freudenthal [7] has considered the physical basis of these processes and their mathematical description, and has not rejected the basic possibility of conversion of the data of one type of test into another. He ascribes the various difficulties in this area to insufficient knowledge about the nature of these processes. Guiu and Pratt [8] have come to the same conclusion and have noted the complexity of the processes taking place in the material under test.

It follows from the above papers that attempts to describe the mechanical behavior of a solid by a single simple equation of state with the same parameters for every type of test are unlikely to succeed. It is clear that the behavior of a solid under different types of test can be described by a generalized equation, but the parameters of this equation will have different numerical values for each individual type of test, reflecting the particular physical conditions prevailing during stress relaxation, creep, and uniaxial strain.

The possibility of describing stress relaxation, creep, and uniaxial strain by a single general equation is discussed below. An analysis of experimental data is used to exhibit both the general and particular features of these processes.

STRESS RELAXATION

Many workers have followed Maxwell and based their studies of stress relaxation phenomena in metals on the assumption that the stress relaxation is proportional to the applied stress, i.e.,

$$-d\sigma / dt = K\sigma, \qquad \sigma = \sigma_0 e^{-Kt}, \qquad (1)$$

where K is a constant representing the rate of spontaneous stress relaxation, and σ_0 and σ are the stresses at times 0 and t.
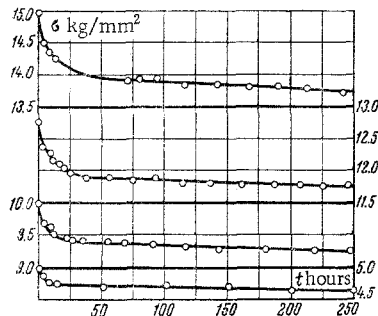

Fig. 1. Stress relaxation data for copper at room temperature obtained with the UMIR-10 machine for different initial stresses σ_0.

It follows that if we plot the stress relaxation curve on a semilogarithmic scale we should obtain a straight line of slope equal to K. However, experimental stress relaxation curves for metals plotted in this way always have a considerable curvature during the initial period of time [9].
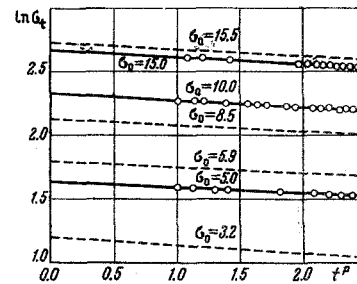

Fig. 2. Stress relaxation curves for copper at room temperature: solid curves were obtained with the UMIR-10 machine; dashed curves were obtained by Davis [14].

Figure 1 shows the stress relaxation data for mark M2 copper obtained at room temperature with the relaxation test machine UMIR-10 designed by Baushis [10]. The specimens were annealed in vacuum at 550° C for 2 hours. These curves were obtained under different experimental conditions as compared with [9] and also take the form of curves rather than straight lines on a semilogarithmic scale. This confirms the qualitative difference noted in [9] between the relaxation process in metals and the viscoelastic process described by the Maxwell system. The accumulated experimental data suggest that Eq. 1 does not reflect the real stress relaxation mechanism in metals.

Belyaev [11] and Baushis [12] have assumed that the stress relaxation process can be described by an equation of the form

$$-d\sigma / dt = K\sigma^n, \qquad (2)$$

whose solution can be written in the form

$$\frac{1}{(n-1)\sigma_0^{n-1}} - \frac{1}{(n-1)\sigma^{n-1}} = -Kt. \qquad (3)$$

Analysis of the relaxation data obtained with the UMIR-10 machine (Fig. 1) on the basis of the equation

$$1/\sigma_0 - 1/\sigma = -Kt,$$

i.e., for n = 2, the more general equation

$$1/\sigma_0 - 1/\sigma = f(t),$$

and for other values of n has shown that Eq. 2 does

not provide a satisfactory description of the stress relaxation process in metals, i.e., K ≠ const.

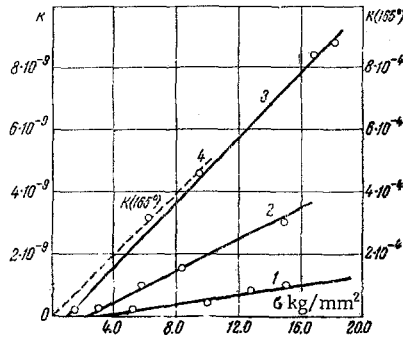Fig. 3. The stress relaxation parameter k as a function of the initial stress σ_0: 1) data obtained with the UMIR-10 machine; 2) Davis' data at room temperature; 3) data from [9]; 4) Davis' data at 165°C.

It was shown in [9] and in a subsequent paper [13] that stress relaxation in metals and alloys at room and higher temperatures can be described by the equation

$$\sigma = \sigma_0 \exp\left[-k^p t^p\right]. \qquad (4)$$

This equation is obtained by integrating Eq. (1) with $K = pk^p t^{p-1}$. It is clear that the time dependence of the stresses has a more complicated character since $K = f(t, \sigma_0)$ and, consequently, the relaxation process in metals is not of the visoelastic type.

The exponent p in Eq. (4) can assume values between 0 and 1 and is a measure of the degree to which the relaxation process in metals approaches the viscoelastic process described by Eq. (1). The parameter k has the dimensions of $[T]^{-1}$ and characterizes the true relative stress-relaxation rate in the body, $1/k = \tau$ is the true relaxation time, and K represents the generalized or effective relative rate of stress relaxation in the solid.
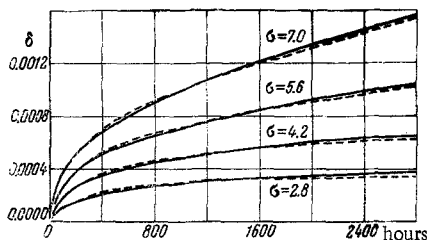

Fig. 4. Creep curves for oxygen-free copper obtained by Davis at 165°C. Dashed curves were obtained by calculation in accordance with Eq. (13).

Figure 2 shows a plot of ln σ_t against t^p based on the experimental data obtained with the UMIR-10 machine and the data reported by Davis [14] who used Boyd's machine to test oxygen-free copper at room temperature. To obtain the graphs of Fig. 2, the experimental data were first plotted in the form of lg L versus lg T, where $L = \ln(\sigma/\sigma_0)$. These graphs gave straight lines whose slope was determined by the parameter p. Having determined this parameter

from the graphs, it was then a simple matter to plot the experimental data in the form of ln σ against t^p. The true character of the process is better exhibited in terms of these coordinates.

The data of Fig. 2 show that Eq. (4). is a good approximation of the stress relaxation process in copper at room temperature. Determinations of the values of p and k have shown that the exponent p for this material is practically independent of the initial stress used in these tests (p = 0.16 and 0.24 for copper at 165°C), while the quantity k increases with decreasing σ_0. It is clear from Fig. 3 that k is a linear function of σ_0. Figure 3 also shows the data for high-purity copper reported in [9] and Davis' data for copper at 165°C [14].

Equation (1) can be obtained from the general Maxwell equation:

$$\frac{d\sigma}{dt} = E\frac{d\varepsilon}{dt} - K\sigma \quad \text{for } \varepsilon = \text{const} \quad (K = \text{const}). \qquad (5)$$

According to Maxwell, there is merely a transition of elastic deformation with time into a new form of deformation, namely, viscous (viscoelastic) deformation, and the initial deformation ε_0 remains constant (pure relaxation). However, Eq. (4) can also be obtained from Eq. (5) when K ≠ const and elastic deformation passes into a plastic deformation: $\varepsilon \rightarrow \delta$, i.e., when the process is elastic-plastic. Equation (5) can therefore be regarded as a generalized equation describing viscoelastic and elastic-plastic processes.
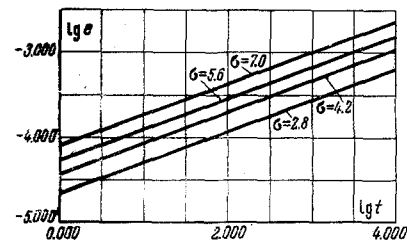

Fig. 5. Creep curves according to Davis [14] using Eq. (13).

When k = 0 (p = 0, $\tau = \infty$), Eq. (5) leads to Hooke's law. After the application of an instantaneous stress to an elastic-plastic body, such that $\sigma_0 < \sigma_e$, where σ_e is the physical elastic limit, the stress relaxation does not occur, and it is only for $\sigma_0 > \sigma_e$ that stress relaxation takes place. The rate of relaxation increases at the same time: $\sigma \rightarrow \sigma_e$, $\varepsilon \rightarrow \varepsilon_e$ (ε_e is the deformation corresponding to the physical elastic limit), and

$$\sigma - \sigma_e = (\sigma_0 - \sigma_e)\exp\left[-k^p t^p\right]. \qquad (6)$$

It is clear that, as the elastic limit increases, there is a reduction in $\sigma - \sigma_e = \sigma^*$ (σ^* is the effective stress) and k. The parameter p is independent of the magnitude of the elastic limit. The data shown in Fig. 3 indicate that the elastic limit of the copper specimen investigated at room temperature is greater than zero, and it is only at 165°C that the elastic limit of copper becomes equal to zero.

CREEP

When σ = const and $K \neq$ const $(0 < p < 1)$ we have from Eq. (5) the equation of creep for an elastic-plastic body:

$$\frac{d\varepsilon}{dt} = \frac{K}{E}\sigma. \qquad (7)$$

Assuming as above that $K = pk^p t^{p-1}$, we have from Eq. (7)

$$\varepsilon = \frac{k^p}{E}\, t^p \sigma. \qquad (8)$$

When $\sigma < \sigma_e$ the entire deformation ε in Eq. (8) is elastic (ε = const) and there is no observable creep, i.e., $\delta = 0$. When $\sigma > \sigma_e$, elastic deformation will continue throughout the time of the test ($\varepsilon = \sigma/E = $ = const), and at the same time there is an increase in the plastic deformation δ: $\varepsilon + \delta = e \neq$ const, while in Eq. (8) we have $\sigma = \sigma_*$. Since deformation reaches an appreciable magnitude during creep, the quantity ε must be interpreted as the true relative deformation under stress e_1, i.e., the Hencky deformation which is related to the Cauchy deformation e_2 by the well-known relation

$$e_1 = \ln (1 + e_2). \qquad (9)$$

Substituting Eq. (9) into Eq. (8) we have, after some rearrangement,

$$1 + e_2 = \exp\left[\frac{k^p}{E} t^p \sigma_*\right]. \qquad (10)$$

If we restrict our attention to the first two terms in the series expansion of this function, we find that the total relative deformation during creep is given by

$$e_2 = \varepsilon + \delta = \frac{k^p}{E}\, t^p \sigma_*. \qquad (11)$$

Figure 4 shows the creep curves (solid lines) for oxygen-free copper obtained by Davis at 165° C [14]. These curves are plotted in the form of lg e_2 versus lg t and can readily be converted into straight lines (Fig. 5). The values of k^p and p ($p = 0.36$ for all the curves) were found by a graphical method, and then Eq. (11) was used to calculate the creep curves shown in Fig. 4 by the dashed lines. The fact that the solid and dashed curves of Fig. 4 are practically identical suggests that the Davis creep curves are satisfactorily described by Eq. (11).
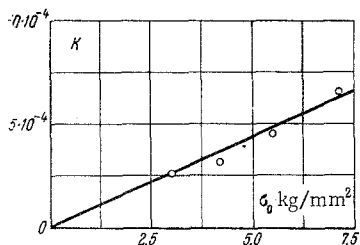

Fig. 6. Stress relaxation parameter k under creep conditions at constantly applied stress (copper at 165° C).

In Fig. 5 the creep curves are shifted not only as a result of the increase in the applied pressure σ_0, but also as a consequence of the attendant increase in k. This is shown by Fig. 6 which gives a plot of k as a function of σ_0. The elastic limit σ_e for copper at 165°C is shown by the data of Fig. 6 to be zero.

UNIAXIAL STRAIN

The uniaxial-strain process is also adequately described by Eq. (5). When a uniaxial stress is applied by constant speed of the test-machine grip, the true relative deformation is $e_1 \neq$ const and $s \neq$ const ($s = P/F$ and $\sigma = P/F_0$, where P is the load, F_0 is the initial cross section, and F the running cross section) and, therefore, $\sigma \approx s$ only for deformations that are not too high.
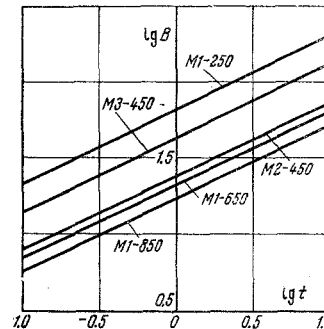

Fig. 7. Uniaxial strain curves for copper, plotted in the form of lg B versus lg t, where B = $= E\delta^{0.5}/\gamma$ using Eq. (18).

As in the above stress relaxation and creep processes we shall assume that $K \neq$ const, i.e., $K = pk^p t^{p-1}$ $(0 < p < 1)$. Substituting into Eq. (5) and assuming in accordance with experiment that $s = \alpha t$, we have, after integration for $s_e > 0$ $(s_e = \sigma_e)$

$$e_1 = \frac{s}{E} + \frac{s_*}{E} k^p t^p. \qquad (12)$$

Putting Eq. (9) into Eq. (12) we obtain

$$1 + e_2 = \exp\left[\frac{s}{E} + \frac{s_*}{E} k^p t^p\right]. \qquad (13)$$

Henceforth we shall restrict our attention to the first two of the expansion terms, so that

$$e_2 = \frac{1}{E} s + \frac{k^p}{E} t^p s_*, \qquad (14)$$

where e_2 is the sum of the elastic and plastic deformations, i.e., $\varepsilon + \delta$.

It was shown earlier [15] that in the case of uniaxial strain in metals

$$e_2 = \varepsilon + \delta = \frac{1}{E} s + \frac{1}{\gamma^2}(s - s_e)^2, \qquad (15)$$

where s_e is the physical elastic limit and γ is the coefficient of plasticity for the material under test. From Eqs. (14) and (15) we find that

$$E\delta^{0.5} / \gamma = k^p t^p. \qquad (16)$$

The length of the specimens tested in [15] was 100 mm and the machine grip was displaced at the rate of 1.2 mm/min. These data and the values of γ and s_e obtained in [15] were used to plot lg B versus lg t,

where $B = E\delta^{0.5}/\gamma$. Since $\lg B = \lg k^p + p \lg t$, it was a simple matter to find the values of p and k^p by graphical methods. The values of p for all the materials were found to be equal to 0.50.

The graphs given in Fig. 7 for copper [15] show that Eq. (16) is a very good representation of the uniaxial strain process.

GENERAL AND SPECIAL FEATURES OF THE PROCESSES

It is clear from the above account that uniaxial strain, creep, and stress relaxation are satisfactorily described by the general equation

$$\frac{ds}{dt} = E\frac{de_1}{dt} - Ks \qquad (K = pk^p t^{p-1}). \qquad (17)$$

Hooke's law ($K = 0$) is obtained from Eq. (17) when $p = 0$. Whe $p = 1$ we have an equation describing the mechanical properties of a viscoelastic body. When $0 < p < 1$ we have an equation describing the mechanical properties of an elastic-plastic body. When $s \neq$ $\neq$ const and $e_1 \neq$ const, Eq. (17) describes uniaxial elastic-plastic strain ($0 < p < 1$). For $s =$ const we obtain the creep equation of Eq. (11), and for $e_1 =$ const we have the condition for stress relaxation in the elastic-plastic body given by Eq. (1).

In pure stress relaxation, the elastic deformation goes over to a generally different deformation, namely, plastic deformation, i.e., $\varepsilon \to \delta$. This is a dislocation process, and since according to [16] for plastic deformation $\delta = (\varepsilon_d^{0.5} - \varepsilon_u^{0.5})$, where ε_d and ε_u are the block sizes (i.e., the sizes of regions responsible for coherent scattering of X-rays) in deformed and undeformed metal, it follows that this process is connected with the fragmentation of blocks or the corresponding increase in dislocation density.

Since $\varepsilon = \sigma/E$, and $\varepsilon + \delta = \varepsilon_0$, we find from Eq. (4) that the increase in plastic deformation with time during the stress relaxation process is given by

$$\delta^{(1)} = \delta / \varepsilon_0 = 1 - \exp[-k^p t^p]. \qquad (18)$$

Here, and in the analysis given below, the superscripts (1), (2), and (3) refer, respectively, to relaxation, creep, and strain processes.

The quantity $\delta^{(1)}$ increases throughout the stress relaxation process from 0 to 1, and the intensity of the process increases with increasing ε_0 (or σ_0), since $k = f(\sigma_0)$. When $0 < \varepsilon_0 \leq \varepsilon_e$ there will be no stress relaxation since then $p = 0$.

According to Davis' data (Fig. 3), the parameter k is temperature-sensitive and appears to depend on the nature of the material. According to the data reported in [13], the exponent p is sensitive to both temperature and material structure.

During creep there is an analogous process of spontaneous transition of elastic deformation into plastic deformation but, in this case, elastic deformation in the body is maintained at a constant level. If we write Eq. (11) in a form analogous to Eq. (18), i.e.,

$$\delta^{(2)} = \delta / \varepsilon = k^p t^p - 1, \qquad (\varepsilon > \varepsilon_e), \qquad (19)$$

we obtain for $\delta^{(2)}$ numerical values of between zero and $\delta^{(2)} =$ const (at the end of the process). During equal time intervals with $\varepsilon^{(2)} = \varepsilon^{(1)}$ we have $\delta^{(2)} \gg$ $\gg \delta^{(1)}$ and, therefore, $p^{(2)} > p^{(1)}$. According to Davis' data (Figs. 3, 6), $k^{(2)}$ is somewhat greater than $k^{(1)}$ (at 165° C) at the same stress.

For uniaxial strain and $\varepsilon_e > 0$ we have from Eq. (14)

$$\delta^{(3)} = \delta / \varepsilon_e = k^p t^p. \qquad (20)$$

The quantity $\delta^{(3)}$ varies between zero and $\delta^{(3)} \gg \delta^{(2)}$ for a given time interval. In the case of uniaxial strain the level of elastic deformation ε increases continuously and, therefore, the rate of stress relaxation is higher than during creep: $k^{(3)} \gg k^{(2)}$ and $p^{(3)} > p^{(2)}$. The parameter k during the strain process is determined by the quantity s_e and, consequently, by the coefficient γ, whose magnitude is also determined by the rate of deformation [16]. The parameter p is equal to 0.50 (for all materials) as shown above (Fig. 7).

Pure stress relaxation, creep, and uniaxial strain are thus essentially processes of increasing plastic deformation due to spontaneous translational factors in the crystallites which make up the polycrystalline materials.

Calculations have shown that experimental data on relaxation, creep, and strain for various types of copper at room temperature and at 165° C are in satisfactory agreement with Eqs. (18), (19), and (20). In general, for a given temperature $p^{(1)} < p^{(2)} < p^{(3)}$, $k^{(1)} \approx k^{(2)}$ and $k^{(2)} \ll k^{(3)}$. It is clear that the parameters k and p, which are quantitative characteristics of the rate of translation phenomena, have different values in different cases, since the way in which the elastic deformation is maintained at each instant of time is different for the above processes.

The possibility of a more precise description of the behavior of materials during creep and stress relaxation in terms of short-period tests involving uniaxial strain is closely related to the problem of finding more accurate quantitative relationships for these parameters. This will be a matter for further study.

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